

Estimating masses of Keplerian disk systems: the case of AGN in NGC 4258

PATRYK MACH, EDWARD MALEC AND MICHAŁ PIRÓG

M. Smoluchowski Institute of Physics, Jagiellonian University, Reymonta 4,
30-059 Kraków, Poland

The Keplerian motion of accretion disks in Active Galactic Nuclei (AGN) is usually believed to be generated by a heavy central mass. We investigate accreting disk systems with polytropic gas in Keplerian rotation and obtain a phenomenological formula that relates the Keplerian angular frequency to the ratio of disk and central masses. Central mass approaches the Keplerian value, if the inner boundary of a disk is close to the minimal stable orbit of a black hole. These results are applied to NGC 4258, the unique AGN with a finely measured Keplerian rotation curve of the central disk, with the conclusion that its rotation curve is, in fact, determined by the central black hole. The mass of the accretion disk exceeds $100M_{\odot}$.

PACS numbers: 95.30.Lz, 95.30.Sf, 98.62.Js, 98.62.Mw

In the astrophysical literature, the velocity of rotation of disks in Keplerian motion is typically associated with a heavy central mass M_c and a light disk mass M_d . The discovery of the massive black hole in the masing nucleus of NGC 4258 exemplifies this interpretation [1]. On the other hand, it is known that even the selfgravity of heavy disks can be compatible with the Keplerian rotation [2]. Huré et al. [3] investigated thin dust disks and pointed out that the rotational Keplerian velocity gives only information on the enclosed mass of the system, and it is not possible to separate masses of the two components — of a disk and a central mass.

We show in this paper that the Keplerian rotation curve, with the frequency ω , does depend on the two masses but they do not contribute additively. A phenomenological formula expresses $M_c G / (r^3 \omega^2)$ (r and G are the cylindrical radius and the gravitational constant, respectively) as a function of $x \equiv M_d / M_c$ and of the ratio $y \equiv r_{\text{in}} / r_{\text{out}}$, where r_{in} and r_{out} are the innermost and outermost disk radii, respectively. If y is small enough then the central mass is reasonably well approximated by the Keplerian value $r^3 \omega^2 / G$.

Consider a stationary, axially symmetric, selfgravitating disk of gas, rotating around a central point mass M_c . The gravitational potential of the system can be written as the superposition $\Phi = -GM_c/|\mathbf{x}| + \Phi_g$. The potential Φ_g is due to the gravity of the disk and satisfies the Poisson equation

$$\Delta\Phi_g = 4\pi G\rho, \quad (1)$$

where ρ denotes the mass density of the gas. In the following, we use the cylindrical coordinates (r, ϕ, z) . The velocity of the gas reads $\mathbf{U} = \omega(r, z)\partial_\phi$. For a barotropic equation of state the frequency ω depends only on the cylindrical radius r [4]. We assume polytropic equations of state $p = K\rho^\Gamma$, where p is the gas pressure, and K and Γ are constant. The Euler equations

$$\nabla p + \rho(\mathbf{U} \cdot \nabla)\mathbf{U} + \rho\nabla\Phi = 0$$

can be integrated, yielding

$$h + \Phi_c + \Phi = C \quad (2)$$

in the region where $\rho \neq 0$. Here, h denotes the specific enthalpy of the fluid $dh = dp/\rho$, and

$$\Phi_c = - \int^r dr' r' \omega^2(r')$$

is the centrifugal potential. The structure of the disk can be obtained from Eqs. (1) and (2) provided that the equation of state and the rotation law $\omega = \omega(r)$ are known.

Let the Keplerian rotation law be $\omega = \omega_0/r^{3/2}$. It follows from Eq. (2) that

$$\omega_0^2 = GM_c + \frac{r_{\text{in}} r_{\text{out}} (\Phi_g(r_{\text{out}}) - \Phi_g(r_{\text{in}}))}{r_{\text{out}} - r_{\text{in}}}, \quad (3)$$

where $\Phi_g(r_{\text{in}}) = \Phi_g(r = r_{\text{in}}, z = 0)$ and $\Phi_g(r_{\text{out}}) = \Phi_g(r = r_{\text{out}}, z = 0)$. We fix r_{out} and the maximal density ρ_{max} . Then, we find that the limit of the right-hand side of Eq. (3) for $r_{\text{in}} \rightarrow 0$ is the Keplerian angular frequency $\omega_0^2 = GM_c$. To show that, note that Φ_g is a bounded function on \mathbb{R}^3 . It is also possible to prove that $\omega_0^2 \rightarrow GM_c$ as $r_{\text{in}} \rightarrow r_{\text{out}}$.

In the numerical analysis we apply the Self-Consistent Field (SCF) scheme [5, 6, 7]. Numerical solutions converge quite well for Keplerian rotation, although the drawback of the method is the need to employ a large number l of Legendre polynomials for thin disks. We find accurate solutions with $l = 400$ and large grids (up to 5000×5000). Such resolutions are permitted because of the simplicity of the SCF method. Our numerical solutions have been tested by the new version of the virial theorem that includes a point mass [8]. It should be pointed out, however, that even $l = 400$

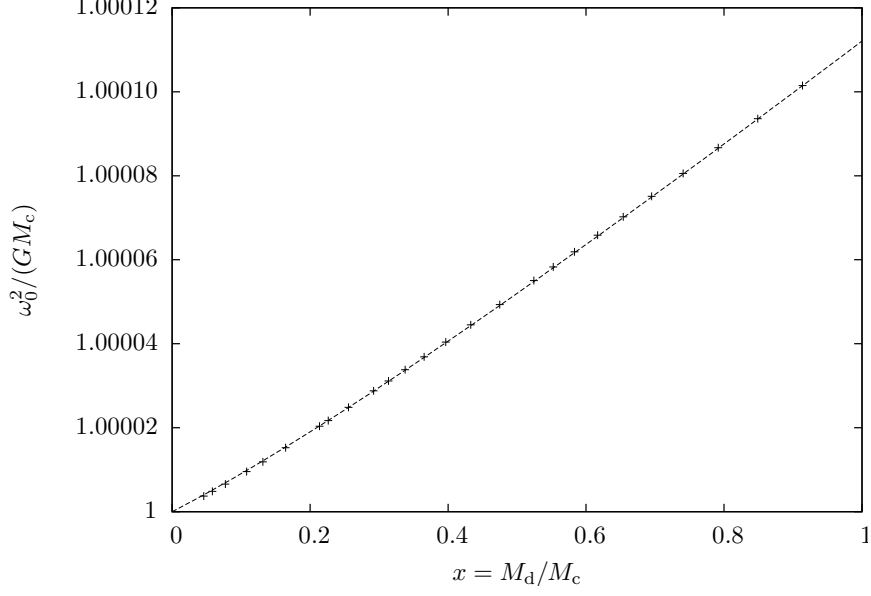


Fig. 1. The dependence of $\omega_0^2/(GM_c)$ on the mass ratio x . The plot is obtained for the adiabatic index $\Gamma = 5/3$ and $r_{\text{in}}/r_{\text{out}} = 10^{-4}$. The dashed line depicts analytic fit (4). Crosses correspond to numerical solutions obtained for different values of M_c .

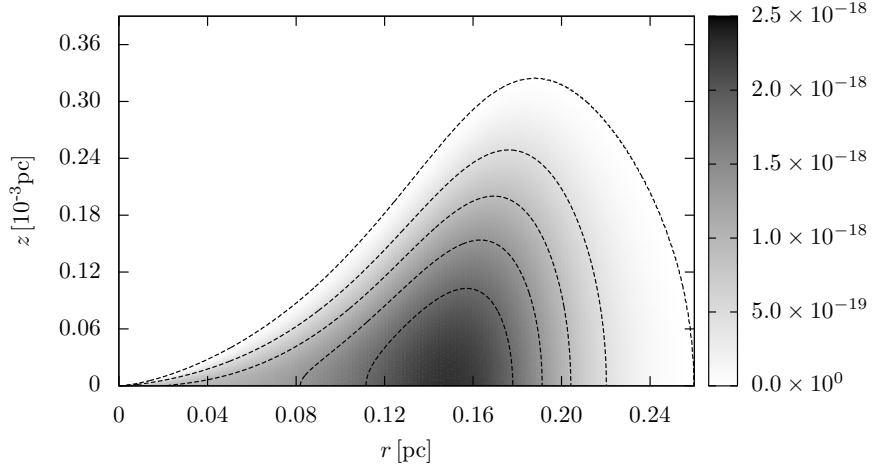


Fig. 2. Mass density profile within a disk model obtained for $\Gamma = 5/3$. The units are g cm^{-3} .

does not suffice to resolve structures in the gravitational potential that are thinner than thinnest disks considered in this paper. We performed tests on thicker disks, demonstrating that important quantities are insensitive to the addition of a higher number of expansion polynomials, which could model fine structures of the gravitational potential within the disk. This conclusion is also supported by the heuristic argument provided by the analysis of infinitely thin disks — they give a regular gravitational potential, i.e., without any structure comparable with the disk thickness.

We found a few thousands of numerical solutions for polytropic disks, mainly with two polytropic indices $\Gamma = 5/3$ (which can be thought to represent monoatomic gas) and $\Gamma = 4/3$ (which might well be associated with vapor water maser disks) and for a few dozens of values of the interior radius corresponding to $y \in (10^{-4}, 0.5)$. Calculations have been done also for a range of polytropic indices in between $4/3$ and $5/3$, and the internal radius corresponding to $y = 10^{-4}$.

Below we shall report results concerning the case $\Gamma = 5/3$, but the essential features of solutions are similar in other cases. The obtained results can be quite well approximated in the quadrant $0 < x < 1$, $10^{-5} < y < 0.55$ by the following phenomenological formula

$$\frac{\omega_0^2}{GM_c} = 1 + xf(y) \frac{g(y) + x}{h(y) + x}, \quad (4)$$

where $x \equiv M_d/M_c$ and f , g and h are defined as follows

$$\begin{aligned} f(y) &= 1.322y - 3.719y^2 + 13.47y^3 - 44.48y^4 \\ &\quad + 116.9y^5 - 206.2y^6 + 208.9y^7 - 90.60y^8, \\ g(y) &= 0.4778 + 0.9495y - 31.67y^2 + 258.4y^3 \\ &\quad - 1431y^4 + 5169y^5 - 11100y^6 + 12760y^7 - 6034y^8, \\ h(y) &= 0.7423 + 0.6313y - 43.57y^2 + 375y^3 \\ &\quad - 2059y^4 + 7281y^5 - 15390y^6 + 17520y^7 - 8242y^8. \end{aligned}$$

Functions f , g and h are nonnegative and smaller than 1 for arguments $0 \leq y \leq 1$.

It is clear from these formulae that the central mass is always smaller than $\omega^2 r^3/G$. At the same time, it is easy to see from (4) that if y is sufficiently small and $M_d < M_c$, then the rotation curve of the gaseous disk is almost exclusively influenced by the central mass.

The number of known AGN's with masers is increasing in recent years [9, 10, 11, 12, 13, 14, 15, 16] (see also for a review [17]), but there is only one case of well resolved Keplerian rotation curve — in the AGN of NGC 4258. Observations of supermasers near the center of the galaxy NGC

4258 [1, 18] provide evidence for a rotating disk of gas surrounding a black hole with a mass $M_c = (3.9 \pm 0.3) \times 10^7 M_\odot$. The motion is Keplerian to high precision, of about a part in a hundred [1, 19, 20, 21], and this mass estimate assumes that velocities are due solely to the gravitational force of the central body. A similar result has been obtained by Siopis et al. [22], who determined the mass of the black hole by constructing axisymmetric dynamical models of the galaxy NGC 4258. Their best mass estimate yields $M_c = (3.3 \pm 0.2) \times 10^7 M_\odot$.

The masers are observed in the annular region that extends from 0.13 pc to 0.26 pc from the center, and the region with detected masers is thin — the maximal relative height is circa $1/800$ (these distances can be expressed in terms of the Schwarzschild radius: $r_g \equiv GM_c/c^2 = 5.8 \times 10^{12} \text{cm}$ and $1 \text{pc} = 5.4 \times 10^5 r_g$). There exists evidence that the disk fills not only the annular region in which masers are detected, but that it extends outwards from a vicinity of the black hole horizon. Reynolds et al. [23] investigated the weakly radiating AGN in NGC 4258 using data from Suzaku, XMM-Newton, and the Swift/Burst Alert Telescope survey. They constrained its luminosity region to between 10 and 4×10^4 Schwarzschild radii of the black hole. In the numerical calculations we assume that the disk extends from $r_{\text{in}} = 2.6 \times 10^{-5} \text{ pc}$ to $r_{\text{out}} = 0.26 \text{ pc}$, and its relative height is equal to $1/800$.

It is clear from (4) — see also Fig. 1 — that if $M_d \leq M_c$ then $\omega_0^2/(GM_c) \approx 1$ with the deviation from unity much smaller than 0.001. The disk's mass, obtained numerically for the polytropic index $\Gamma = 5/3$ and assumed geometrical parameters of the disk, reads $M_d = 2.62 \times 10^{-6} M_c \approx 102 M_\odot$.

The results for other polytropes, $5/3 > \Gamma \geq 4/3$, suggest that disk's mass depends rather weakly on the equation of state and increases (up to $M_d = 190 M_\odot$) with the decrease of the polytropic index. The disk's size can be larger than that detected in observations, which, in turn, would yield a larger mass. Therefore $M_d = 100 M_\odot$ can be regarded as a conservative lower bound on the mass of the gaseous disk in the AGN of NGC 4258. A sample distribution of the mass density within the disk, obtained for $\Gamma = 5/3$, is shown on Fig. 2.

The above results are consistent with those rough mass estimates that base on such values of the baryonic number density ρ_n within the disk that favor the existence of water masers. If one assumes that ρ_n is around 10^{10} per cm^3 , then the mass is around $10^4 M_\odot$ [1, 18]. Herrnstein et al. [21] note that the allowed density is $10^8/\text{cm}^3 < \rho_n < 10^{10}/\text{cm}^3$; the lower bound gives the disk mass $\approx 100 M_\odot$.

This analysis is entirely Newtonian, and since it applies to systems with black holes, it should be repeated in the general-relativistic context. We do not expect essential changes concerning those systems, where the inner disk

boundary is well separated from a black hole horizon, $r_{\text{in}} \gg 6M_{\text{c}}$.

Acknowledgments

The research was carried out with the supercomputer “Deszno” purchased thanks to the financial support of the European Regional Development Fund in the framework of the Polish Innovation Economy Operational Program (contract no. POIG.02.01.00-12-023/08).

REFERENCES

- [1] M. Miyoshi, J. Moran, J. Herrnstein, L. Greenhill, N. Nakai, P. Diamond, I. Makoto, *Nature*, **373**, 127 (1995)
- [2] M. Hashimoto, Y. Eriguchi, E. Müller, *A&A*, **297**, 135 (1995)
- [3] J.-M. Huré, F. Hersant, C. Surville, N. Nakai, T. Jacq, *A&A*, **530**, 145 (2011)
- [4] J.L. Tassoul, *Theory of Rotating Stars*, Princeton Univ. Press, Princeton, NJ 1978
- [5] J.P. Ostriker, J.W-K. Mark, *ApJ*, **151**, 1075 (1968)
- [6] M.J. Clement, *ApJ*, **194**, 709 (1974)
- [7] S.I. Blinnikov, *Soviet Astron.*, **19**, 151 (1975)
- [8] P. Mach, *MNRAS*, **422**, 772 (2012)
- [9] J. Braatz, L. Greenhill, J. Moran, A. Wilson, J. Herrnstein, *Bulletin of the American Astronomical Society*, **29**, 1374 (1997)
- [10] L.J. Greenhill, J.M. Moran, J.R. Herrnstein, *ApJ*, **481**, L23 1997
- [11] L.J. Greenhill, C.R. Gwinn, *ApSS*, **248**, 261 (1997)
- [12] A.S. Trotter, L.J. Greenhill, J.M. Moran, M.J. Reid, J.A. Irwin, K.-Y. Lo, *ApJ*, **495**, 740 (1998)
- [13] A. Yamauchi, N. Nakai, N. Sato, Ph. Diamond, *PASJ*, **56**, 605 (2004)
- [14] P.T. Kondratko, L.J. Greenhill, J.M. Moran, *ApJ*, **618**, 618 (2005)
- [15] Y. Ishihara, N. Nakai, N. Iyomoto, K. Makishima, Ph. Diamond, P. Hall, *Publications of the Astronomical Society of Japan*, **53**, 215 (2001)
- [16] L.J. Greenhill, P.T. Kondratko, J.E.J. Lovell, T.G.H. Kuiper, J.M. Moran, D.L. Jauncey, G.P. Baines, *ApJ*, **582**, L11 (2003)
- [17] P.T. Kondratko, L.J. Greenhill, J.M. Moran, *ApJ*, **678**, 87 (2008)
- [18] J. Moran, L. Greenhill, J. Herrnstein, P. Diamond, M. Miyoshi, N. Nakai, M. Inque, *PNAS*, 9211427 (1995)
- [19] E. Maoz, *ApJ*, **447**, L91 (1995)
- [20] J.M. Moran, 2008, Frontiers of Astrophysics: A Celebration of NRAO’s 50th Anniversary ASP Conference Series, Vol. 395, Proceedings of the conference held 18–21 June, 2007, at the National Radio Astronomy Observatory, Charlottesville, Virginia, USA

- [21] J.R. Herrnstein, J.M. Moran, L.J. Greenhill, A.S. Trotter, *ApJ*, **629**, 719 (2005)
- [22] Ch. Siopis, K. Gebhardt, T.R. Lauer, J. Kormendy, J. Pinkney, D. Richstone, S.M. Faber, S. Tremaine, M.C. Aller, R. Bender, *ApJ*, **693**, 946 (2009)
- [23] Ch.S. Reynolds, M.A. Nowak, S. Markoff, J. Tueller, J. Wilms, A.J. Young, *ApJ*, **691**, 1159 (2009)